Lesson Plan: Elasticity

Background

Many materials and substances can return to their original shape and size after being pulled on or twisted. This property is called **elasticity**. This is also a property of substances that make up the human body – an example is a strand of hair – as well as materials made by humans. Elasticity causes our skin to stretch when pulled or pinched, and then go back to its original shape, and it also explains why rubber balls bounce. Cellular components, such as the mitotic spindle which is present during division, have also been shown to have elastic properties – for example, the mitotic spindle can regain its shape after partial squashing.

The ability of materials or substances to undergo a reversible increase in length, similar to a spring, is a property that can be measured. This property is known as the elastic modulus or **Young's modulus.** This lesson plan demonstrates the elasticity of biological materials, as well as materials made by humans. Students will measure changes in length of materials under normal conditions and under conditions that cause changes in their elasticity.

Objectives & Grade Level

Demonstrate elasticity of biological materials and those made by humans, and measure changes in length with different forces. Determine the effects of temperature and water (hydration) on length changes. Advanced students will be able to calculate Young's modulus for different materials. Appropriate for middle school to advanced high school science classes; see notes for advanced students.

Materials

- Strand of hair (~15-25 *cm* = ~6-10 inches long)
  Use a strand of your own hair or ask a classmate for one. If asking a classmate, be sure to obtain permission first and let him or her provide you with a strand of hair, rather than taking it yourself
- Silk thread (~15-25 *cm* = ~6-10 inches long)
- Rubber band
- Other materials (String, Rope, Cord, Wire) can also be used
- Ruler
- Rod to suspend materials
- Hooks to attach weights
  Paper clips can be used instead of hooks
- Weights (~100 *g*, ~200 *g*, ~300 *g*, ~400 *g*)
  Weights can be made using small vials or bottles filled with water or nails; be sure to measure the exact mass of each weight before use

Procedure

1. Take a single strand of hair, a piece of silk thread and a rubber band, and examine their overall elastic properties. Materials are elastic only up to a certain pulling force, after which they will not return to their original shape. This point is called the **elastic limit**. Increasing the force even more will cause the material to break – this is called the **breaking point**. Try stretching each material by pulling on them gently – don't pull too hard or they will not return to their original shape, or break!

Which of the materials do you think is the most elastic? Which do you think is the least elastic?

2. Now soak the strand of hair and silk thread in water for 5-10 minutes and put the rubber band in the freezer or on ice for 5-10 minutes. Try stretching the wet hair and wet silk thread; then try stretching the cold rubber band.

Are the wet hair and wet silk thread still elastic? Are they harder or easier to stretch than they were when they were dry? What about the cold rubber band – is it harder or easier to stretch than when it was at room temperature?

What do you think the effects of soaking in water (this is called 'hydration') are on the elasticity of hair and silk? What about the effects of cold temperature on the elasticity of a rubber band?

3. We can measure the elasticity of each material by seeing how much it stretches when different weights are added. When doing these experiments, we want to measure each material within its linear elasticity range, which is the range before you reach the elastic limit.

Don't add a weight that is too large – you may reach the elastic limit of the material, or even its breaking point!

4. Attach the strand of hair to a rod and measure its starting length with a ruler. Write down your measurement. Then measure the starting length of the silk thread and rubber band in the same way. Don't forget to write down your measurements!

The hair and silk thread can be tied or taped at the ends, then looped over a rod – measure from the edge of the knot or tape to the bottom of the loop to find the starting length; straighten the loop and make sure both sides are the same length before you do the measurement. The rubber band can just be looped over a rod, as shown in ***Figure 1****, and measured.*

5. Attach a weight of ~100 *g* to the bottom of the strand of hair, as shown for the rubber band in **Figure 1**. Make sure that the weight extends the strand as much as it can without your pulling on it! Measure the length of the strand of hair with the weight and write it down.

Has the strand of hair increased in length with the weight? If the strand is longer, how much did it increase in length? If the strand is the same length, try a heavier weight.

6. Change the weight to ~200 *g*, then to ~300 *g* and ~400 *g*, and repeat your length measurements.

Be sure to write down the exact mass of the weight you used, together with your measurements .

7. Now try the same measurements, but under conditions that affect the elasticity of the materials. Soak the strand of hair and silk thread in a small container of water for 5-10 minutes, then repeat the measurements with each of the weights.

Don't let the strand of hair or silk thread dry out between measurements! Soak them again in water for a few minutes between each measurement, if necessary.

Are the lengths (with the weights) of the wet hair strand and silk thread the same as for the dry hair strand or silk thread? If they are different, are the lengths longer or shorter for wet hair and wet silk thread?

8. Put the rubber band in a freezer or on ice for 5-10 minutes. Then repeat the measurements with each of the weights and record your measurements.

Don't let the rubber band warm up between measurements! Freeze it again or put it on ice for a few minutes between each measurement to keep it cold.

Are the lengths (with the weights) of the cold rubber band the same as for the rubber band at room temperature? If the lengths are different, are they greater or smaller than for the rubber band at room temperature?

9. Advanced students will be able to calculate the Young's Modulus for each material and difference in conditions, as follows:

a) First, calculate the force, F, for each of the weights that you used in your measurements using the exact mass of each weight and Newton's second law:

$$F = ma$$

where
- F is the applied force
- m is the mass of the weight you added
- a is acceleration due to gravity = 9.81 m/s^2

and then convert to Newtons (N), which is the same as $kg{\cdot}m/s^2$.

The example shown below is for a weight with a mass of 100 g:

$$F = ma$$

$$= 100\ g \times 10^{-3}\ kg/g \times 9.81\ m/s^2$$

$$= 0.981\ kg{\cdot}m/s^2 \text{ or } N$$

b) Now determine the cross-sectional area, A, of the single strand of hair, silk thread and rubber band that you used in your experiments. To do this, measure the diameter in mm as accurately as you can, then divide by 2 to find the radius and use the equation for the area of a circle ($A=\pi r^2$) to find the cross-sectional area. If you are using a rubber band that has a rectangular cross-section, find the area after measuring the width, *w*, and height, *h,* of the rectangle ($A=w\,h$). The area units should be mm^2.

Human hair can range from ~0.02 to 0.18 mm in diameter. If you are having trouble measuring the diameter of the strand of hair, try using an average value of 0.08 mm for the diameter instead. The diameter of a silk thread also varies, but the range is ~0.01-0.025 mm in diameter. Try using an average value of 0.02 mm for the diameter of your silk thread, if it is too small for you to measure it accurately. The rubber band shown in Figure 1 has a cross-sectional width of 1 mm and length of 5 mm – its cross-sectional area is 5 mm^2*.*

c) Now find *F/A* by dividing the force for each of your weights by the cross-sectional area of the material. Since the weight is pulling on both sides of the rubber band, the total cross-sectional area for both sides is 2 x 5 mm^2, or 10 mm^2. If you made loops of the strand of hair and silk thread and put the weights at the bottom of the loop, you should multiply the cross-sectional area for each strand by 2 to find the total area. After you divide the force corresponding to each weight by the cross-sectional area of the material to find *F/A* , the units will be N/mm^2, which is the same as *milli-Pascals* (*mPa*).

d) For each weight, you should also find a value for $\Delta L/L_0$

where

- ΔL is the change in length with the force (subtract the starting length from the length with the weight)
- L_0 is the starting length

e) For each material, make a table like the one shown below – be sure to insert the exact mass for each weight rather than the approximate ones shown:

Material: Hair strand	Mass of Weight	*F*	*F/A*	*L* (with *F*)	ΔL ($=L-L_0$)	$\Delta L/L_0$
A=	0					
L_0=	~100 *g*					
	~200 *g*					
	~300 *g*					
	~400 *g*					

f) Plot your values for *F/A* and $\Delta L/L_0$ for each material to obtain a graph like the one shown in **Figure 2**. The quantity *F/A* is also known as **stress** and $\Delta L/L_0$ is referred to as **strain** (see **Note 3** below). You should have one graph for each of the materials that you measured.

The slope of the initial, linear portion of the line is the Young's Modulus, *E*, which is also defined as

$$E = \frac{F/A}{\Delta L/L_0}$$

where

- *F* is the applied force
- *A* is the cross-sectional area
- ΔL is the change in length with the force (subtract the starting length from the length with the weight)
- L_0 is the starting length

g) Find the slope by calculating the change in *y* divided by the change in *x*, or $\Delta y/\Delta x$, from your graphs.

How do your values for Young's modulus compare with those in the table in ***Note 2****?*

The Young's modulus values are smaller for materials that are more elastic than those that are less elastic. Is wet hair or silk more elastic or less elastic than dry hair or silk? What about cold rubber compared to rubber at room temperature?

Notes

1. Hair and silk are biological materials that show elasticity. Hair is a long polymer made of a protein called keratin, which is also called an intermediate filament protein. Silk is made by silkworms from a protein known as fibroin. Hair and silk both have high tensile strength – they can be pulled on with high force before breaking. The elasticity of hair varies with individuals, while the elasticity of silk thread varies with the way the silk is made into thread.

Rubber bands are also made by humans, although most rubber bands are made from natural rubber, which is a milky fluid produced by rubber trees. Rubber bands and other things made from rubber have a very high elasticity.

2. External conditions can change elasticity of materials. For example, wet hair shows higher elasticity than dry hair – the Young's modulus is lower for wet hair than dry hair. Does your hair seem longer after combing when wet? It's because it actually is longer! Rubber is unusual in that it becomes more elastic at cold temperatures (down to a certain point) than warmer ones. This is reflected in a lower Young's modulus at cold temperatures.

The table below shows approximate values for Young's modulus and changes under different conditions; values for hair are given at two different conditions of relative humidity (RH).

	Young's Modulus (Approximate values)	Young's Modulus Value Changes
Strand of hair	5,394 *MPa* (65% RH)	Decreases with hydration e.g., 2,059 *MPa* (100% RH)
Silk (silkworm)	8,900 *MPa*	Decreases with hydration
Rubber	0.05 *MPa* - 2 *MPa*	Decreases with temperature

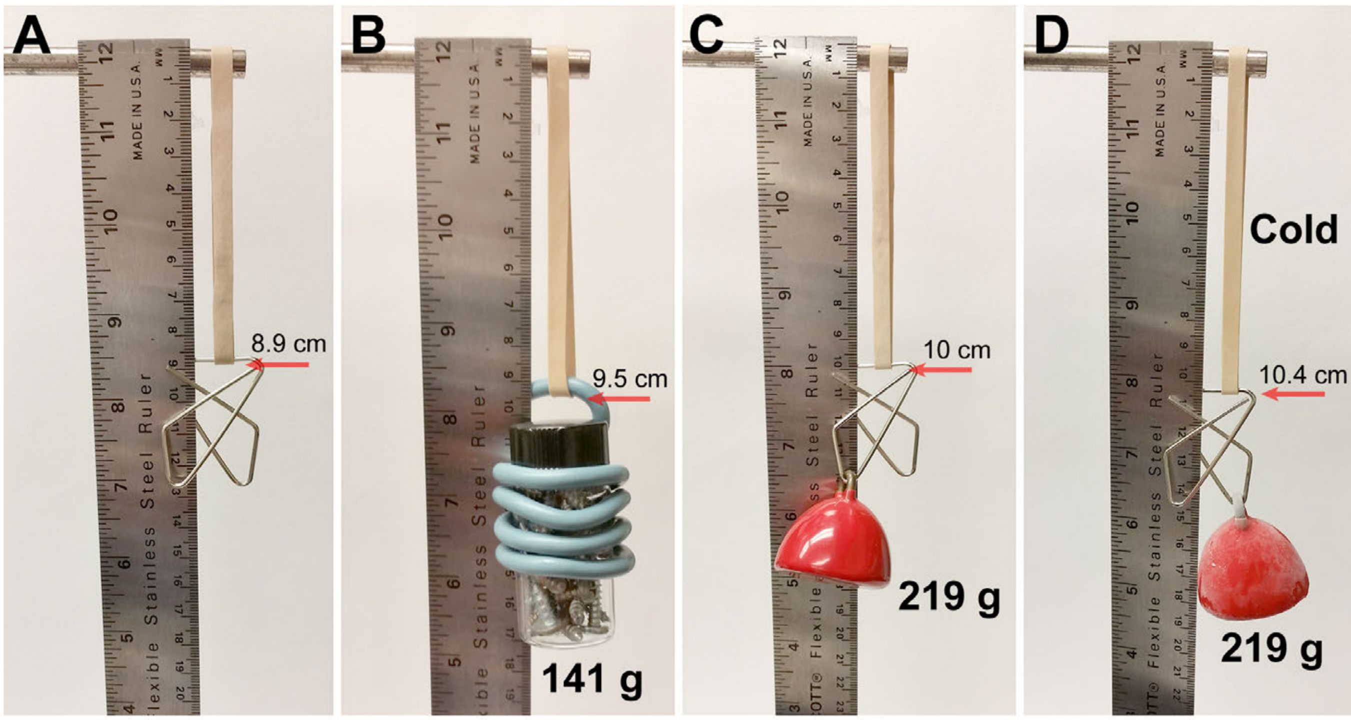
3. **Advanced topic 1**: measurements of the Young's modulus of materials take into account two quantities, stress and strain. **Stress** is the force causing the material to change, *F/A*. **Strain** is the measure of deformation of the materials, or $\Delta L/L_0$. Young's modulus is the ratio between stress and strain, and can be determined by plotting stress vs strain, and determining the slope of the initial, linear part of the curve, as described in step 9 above. Young's modulus only applies to the linear region of the curve.

4. **Advanced topic 2**: Young's modulus can differ when force is applied in different directions along a material. For example, wood varies in elasticity depending on whether the forces are along the grain of the wood or not. This is also true of many other materials.

5. **Advanced topic 3**: **Hooke's Law** states that the force acting on a spring to compress or extend it by a given distance is proportional to the distance. Hooke's Law is given as $F=-kx$, where F is force, κ is the spring constant and x is distance (or length). It assumes an elastic and linear response. Elastic materials follow Hooke's Law in their initial, linear response to force, behaving like a spring.

Figures

Figure 1 Measuring Elasticity of a Rubber Band



A) The starting length of the rubber band was 8.9 *cm*. The force due to the paper clip (0.0343 *N* = 3.5 *g* x 9.81 m/s^2 x 10^{-3} *kg/g*) is too small to change the rubber band's length. **B)** A weight (made from a small bottle filled with screws and wrapped in a flexible lead coil) increased the length to 9.5 *cm*. **C)** A heavier weight caused the rubber band to increase to 10 *cm* in length**. D)** Cooling the rubber band caused it to increase in length even more with the same weight as in **C**.

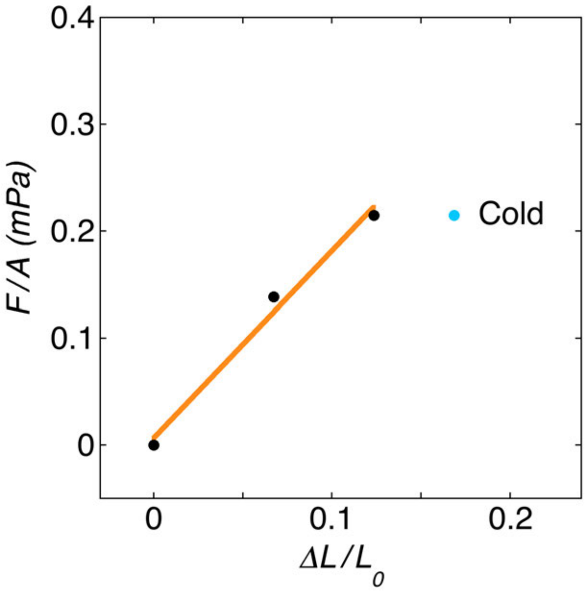


Figure 2 Determining Young's Modulus from a Plot of Stress vs Strain

Plotting stress (*F/A*) vs strain ($\Delta L/L_0$) and fitting the points to a line gave a value for Young's modulus of *E*=1.75 *mPa* for the rubber band shown above. If you have more points in your plots, the ones corresponding to higher stress and strain may not fall onto the line. In this case, using only the initial points, draw a line that goes through or is close to as many points as possible, and determine Young's modulus from the slope of the line. The single point (blue) shown for the cold rubber band does not fit the orange line very well – this indicates that the point differs from the others. Young's modulus calculated from this point is *E*=1.27 *mPa*. The longer length of the cold rubber band (see **Figure 1**) and lower value for *E* than at room temperature indicates that the elasticity of the rubber band has increased with cold.

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